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LETTER TO THE EDITOR

Thermal conductance and Lorentz number for fractional exclusion particle

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Abstract. The thermal conductance and the Lorentz number for fractional exclusion particles are investigated in the ballistic regime. By using Ramo–Shockley theory, it is shown that the thermal conductance is independent of the statistical properties under the particle complete degeneracy condition, but the Lorentz number is not, and they are both independent of the transport material.

In recent years, investigations of quantum transport in material have uncovered many interesting types of behaviour in non-equilibrium [1, 2]. In this respect, a theoretical study of the thermal conductance of a charged-particle carrier and a phonon carrier has been made [3–5]. These results show that in a one-dimensional transport wire the thermal conductance has a universal unit $K^2\pi^2/3h$ (K is the Boltzmann constant and h is the Planck constant), if the transport happens in the carrier ballistic regime. By using the linear responding theory, some authors [6–8] have calculated the electronic thermal conductance and the Lorentz number for one-dimensional ballistic transport, and provided fundamental properties of the physical quantities. They naturally obtained the universal thermal conductance for the particle degenerate case. In [9, 10], by using Haldane's concept of fractional exclusion statistics and the Laudauer formulation [11, 12] of transport theory, it has been shown that the thermal conductance is independent of the statistics in one-dimensional ballistic transport in the completely degenerate case.

The aim of this letter is to use Ramo–Shockley theory [13] to present the conductance and the Lorentz number for fractional statistical particles in a one-dimensional ballistic transport wire by using the relaxation time model. We will see that the Lorentz number depends on the particle's statistical properties through the parameter g, but the thermal conductance does not.

Referring to Ramo–Shockley theory [14], the total current operators $I_\mu(t)$ can be expressed as

$$I_n(t) = \frac{1}{L}\int_{-L/2}^{L/2} \mathrm{d}z \int_{-\infty}^{\infty} \mathrm{d}k\, \epsilon_k^n \frac{\hbar k}{m} g(k, z, t) \qquad (n = 0, 1) \tag{1}$$

where $I_0(t)$ and $I_1(t)$ represent the particle and energy current, respectively. L is the wire length, ϵ_k is the kinetic energy of the particle and $g(k, z, t)$ represents the Wigner function.

We define the auto-correlation function

$$C_{n,l}(t) = \tfrac{1}{2}\langle \delta I_n(0) \cdot \delta I_l(t) + \delta I_n(t) \cdot \delta I_l(0)\rangle \qquad (n, l = 0, 1) \tag{2}$$

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where the fluctuation of the current is $\delta I_n(t) = I_n(t) - \langle I_n(t)\rangle$. Substituting the current (1) into the auto-correlation function (2), we have

$$C_{n,l}(t) = \frac{1}{2}\left(\frac{\hbar}{mL}\right)^2$$
$$\int_{-L/2}^{L/2} \mathrm{d}z \int_{-L/2}^{L/2} \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}k \int_{-\infty}^{\infty} \mathrm{d}k'\, kk'\, \epsilon_k^n\, \epsilon_{k'}^l [\delta f(k', z', 0; k, z, t) + \delta f(k, z, 0; k', z', t)] \quad (3)$$

where the fluctuation function

$$\delta f(k', z', 0; k, z, t) = \langle g(k', z', 0) g(k, z, t)\rangle - \langle g(k', z', 0)\rangle\langle g(k, z, t)\rangle.$$

In the relaxation time model the fluctuation takes the form [14]

$$\delta f(k', z', 0; k, z, t) = \delta f\left(k', z', 0; k, z - \frac{\hbar k t}{m}, 0\right) \exp\left(-\frac{t}{\tau_c}\right) \quad (4)$$

where τ_c is the relaxation time. It turns out that the problem consists of the calculation of $\delta f(k', z', 0; k, z - (\hbar k t/m), 0)$. As in the case of the Bose–Einstein and Fermi–Dirac distributions, the fluctuation of fractional particles is given by

$$\delta f(k', z', 0; k, z, 0) = \frac{1}{\pi}\frac{\partial \eta}{\partial x}\delta(k - k') N(k, z, z') \quad (5)$$

where $N(k, z, z')$ is an arbitrary normalized function in the system. This function describes the space correlation at equal times between the point z and z' of the state k. The interesting result is that it plays no role in the auto-correlation function, i.e. $\int_{-L/2}^{L/2} N(k, z, z')\,\mathrm{d}z' = 1$. As a consequence, the final result should be independent of the assumption of the coherent length. The distribution function η of the fractional particle in (5) is

$$\eta(x, g) = \frac{1}{W(x, g) + g} \quad (6)$$

where $x = (\epsilon - \mu)/KT$, K is the Boltzmann constant, and T and μ are the temperature and the chemical potential, respectively. The function $W(x, g)$ satisfies the general equation

$$W^g(x, g)[1 + W(x, g)]^{1-g} = \mathrm{e}^x. \quad (7)$$

Obviously $g = 0$ corresponds to the Bose–Einstein distribution and $g = 1$ corresponds to the Fermi–Dirac distribution. Combining (3) with (4), the auto-correlation function is written as

$$C_{n,l}(t) = \frac{1}{\pi}\left(\frac{\hbar}{mL}\right)^2 \mathrm{e}^{t/\tau_c} \int \mathrm{d}z\, \mathrm{d}z'\, \mathrm{d}k\, k^2 \epsilon_k^{n+l} \frac{\partial \eta}{\partial x} N\left(k, z - \frac{\hbar k t}{m}, z'\right)$$
$$= \frac{1}{\pi}\left(\frac{\hbar}{mL}\right)^2 \mathrm{e}^{t/\tau_c} \int_{-L/2}^{L/2} \mathrm{d}z \int_{(m/\hbar t)(z-(L/2))}^{(m/\hbar t)(z+(L/2))} \mathrm{d}k\, k^2 \epsilon_k^{n+l} \frac{\partial \eta}{\partial x}$$

where m is the mass of the particle. Since the integrated function in the above expression is independent of the argument z, we exchange the integral arguments z and k, and then have

$$C_{n,l}(t) = \frac{2}{\pi}\left(\frac{\hbar}{mL}\right)^2 \mathrm{e}^{t/\tau_c} \int_0^{mL/\hbar t} \mathrm{d}k\, k^2 \epsilon_k^{n+l} \frac{\partial \eta}{\partial x}\left(L - \frac{\hbar t}{m} k\right).$$

Using the relation between the momentum and the energy of the particle $\epsilon = \hbar^2 k^2/2m$, the auto-correlation function $C_{nl}(t)$ can be expressed as

$$C_{nl}(t) = \frac{2^{3/2}}{\pi \hbar L m^{1/2}} \mathrm{e}^{t/\tau_c} \int_0^{L^2 m/2t^2} \mathrm{d}\epsilon\, \epsilon^{n+l+1/2} \frac{\partial \eta}{\partial x}\left(1 - \frac{t}{L}\left(\frac{2\epsilon}{m}\right)^{1/2}\right). \quad (8)$$

In order to give the thermal conductivity, the noise spectral density should be calculated. First it is noted that

$$S_{n,l}(\omega) = q \int_{-\infty}^{\infty} \exp(\mathrm{i}\omega t) C_{n,l}(t)\, \mathrm{d}t \tag{9}$$

where q is the degeneracy degree of the carriers. Our concern is the static spectral density. Due to the explication of the integrated function for ϵ, we first integrate the argument t in the ballistic regime, $t/\tau_c \ll 1$, which leads to

$$S_{n,l}(0) = \frac{q}{h} \int_0^{\infty} \mathrm{d}\epsilon\, \epsilon^{n+l} \frac{\partial \eta(x,g)}{\partial x} = \frac{qKT}{h} \int_{x_0}^{\infty} \mathrm{d}x\, (KTx + \mu)^{n+l} \frac{\partial \eta(x)}{\partial x} \tag{10}$$

where $x_0 = -\mu/KT$. By using (6), we can calculate the function

$$\frac{\partial \eta(x)}{\partial x} = -\frac{W(x)[1 + W(x)]}{(g + W(x))^3}.$$

After some calculation it turns out that when $g = 0$ or 1 the integral for $S_{\mu\nu}(0)$ is easy to perform and gives the same result as that in [3–5], but for the fractional particles we are unable to integrate directly. For simplicity, we first consider the completely degenerate case in which $\mu/KT \to \infty$. We note that $\lim_{w\to 0} x = -\infty$ for $g \neq 0$ and $\lim_{w\to\infty} x = \infty$ for any $g \geqslant 0$. This shows that the integral for x can be replaced by an integral over W and the integral space is from 0 to ∞, i.e.

$$S_{n,l}^{deg}(0) = \frac{qKT}{h} \int_0^{\infty} \mathrm{d}W\, (KTx + \mu)^{n+l} \frac{1}{(W + g)^2} \tag{11}$$

where $x = \ln(W + 1) + g(\ln W - \ln(W + 1))$. After integration the results are

$$S_{00}^{deg}(0) = \frac{qKT}{hg} \qquad S_{01}^{deg}(0) = S_{10}^{deg}(0) = \frac{\mu q KT}{hg} \qquad S_{11}^{deg}(0) = \frac{qKT}{h}\frac{\pi^2}{3} + \frac{\mu^2 qKT}{gh}. \tag{12}$$

The kinetic coefficients are $L_{n,l}(\omega) = LS_{n,l}(\omega)/(KT)$ and the thermal conductivity is

$$\kappa = \frac{1}{T}\left[L_{11}(0) - \frac{L_{10}(0)L_{01}(0)}{L_{00}(0)}\right].$$

Putting the above results together, we obtain

$$\kappa = \frac{K^2\pi^2}{3h} qLT. \tag{13}$$

Furthermore, the thermal conductance in the ballistic regime is $\kappa' = \kappa/L = (K^2\pi^2/3h)qL$. It should be noted that if the particles are charged, the electric current should be the particle current multiplied by the charge carried by the carriers, i.e. S_{00} should be multiplied by e^2 and $S_{01} = S_{10}$ by e, but S_{11} is left unchanged. It is readily seen that the thermal conductivity does not change for the charged carrier, and the electric conductance in the ballistic regime is $\sigma(0) = e^2 L_{00}(0)/L = e^2 q/hg$. This shows that the Lorentz shape number takes the form $L_0(0) = \kappa'/\sigma(0)T = (\pi^2 K^2/3e^2)g$, which depends on the statistical properties of the carriers through parameter g.

In conclusion, a rigorous calculation of the thermal conductance for the fractional exclusion particle in one-dimensional ballistic transport has proved that in complete degeneracy this physical quantity is independent of the statistical properties through the parameter g, but the Lorentz number is not. We also show that both of the quantities are independent of the characteristics of the transport material. If $g = 1$ is selected, our conclusion coincides with Fermion-carrier case.

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